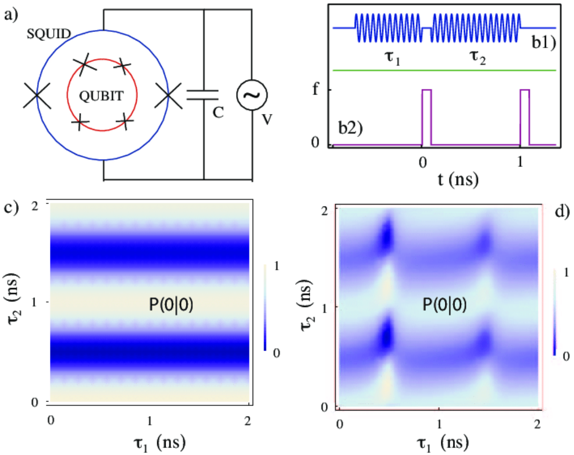
Superconducting-flux qubits



4-Josephson-junction-superconducting-flux qubit

Superconducting-flux qubits

Unlike microscopic entities-electrons, atoms, ions, and photons- on which other qubits are based, superconducting quantum circuits are based on the electrical (LC) oscillator and are macroscopic systems with a large number of (usually aluminum) atoms assembled in the shape of metallic wires and plates.

Superconducting qubits operates through the following phenomena: superconductivity, which is the frictionless flow of electrical fluid through the metal at low temperature (below the superconducting phase transition),

and the Josephson effect, which endows the circuit with nonlinearity without introducing dissipation or dephasing.

The collective motion of the electron fluid around the circuit is described by the flux ϕ threading the inductor, which plays the role of the center-of-mass position in a mass-spring mechanical oscillator.

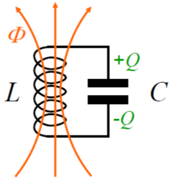
We will show how a Josephson tunnel junction transforms the circuit into a true artificial atom, for which the transition from the ground state to the excited state ($|g\rangle$-$|e\rangle$) can be selectively excited and used as a qubit, unlike in the pure LC harmonic oscillator.

Classical LC Harmonic Oscillator

We start with the classical description of a linear LC resonant circuit, where energy $E(t)$ oscillates between electrical energy in the capacitor C and magnetic energy in the inductor L.
The instantaneous, time-dependent energy in each element is derived from its current and voltage,

$$E(t) = \int_{-\infty}^{t} V(t') \, I(t') \, dt' \tag{1}$$

where $V(t')$ and $I(t')$ denote the voltage and current of the capacitor or inductor.



Classical LC Harmonic Oscillator

Here, we represent the circuit elements in terms of one of its generalized circuit coordinates, charge $Q(t)$ or flux $\Phi(t)$ defined as the time integral of the voltage:

$$\Phi(t) = \int_{-\infty}^{t} V(t')\, dt' \tag{2}$$

By using the relations $V = LdI/dt$ and $I = CdV/dt$ and applying the integration by parts formula, we can write down energy terms for the capacitor and inductor in terms of the node flux:

$$\mathcal{T}_C = \frac{1}{2} C \dot{\Phi}^2 \tag{3}$$

$$\mathcal{U}_L = \frac{1}{2L} \Phi^2 \tag{4}$$

Classical Lagrangian and Hamiltonian

The Lagrangian can be written as the difference between the kinetic and potential energy terms:

$$\mathcal{L} = \mathcal{T}_C - \mathcal{U}_L = \frac{1}{2}C\dot{\Phi}^2 - \frac{1}{2L}\Phi^2 \tag{5}$$

By using the Legendre transformation, we need to calculate the momentum conjugate to the flux, which in this case is the charge on the capacitor:

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi} \tag{6}$$

The Hamiltonian of the system is now defined as:

$$H = Q\dot{\Phi} - \mathcal{L} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \equiv \frac{1}{2}CV^2 + \frac{1}{2}LI^2 \tag{7}$$

Note that this Hamiltonian has the same mathematical structure of a mechanical harmonic oscillator which can be expressed in position, and momentum (x, p) coordinates in the form:

$$H = p^2/2m + m\omega^2 x^2/2$$

Quantum LC Hamiltonian

A quantum-mechanical description of the system can be obtained by expressing the charge and flux coordinates as quantum operators. The classical coordinates satisfy the Poisson bracket:

$$\{f, g\} = \frac{\delta f}{\delta \Phi}\frac{\delta g}{\delta Q} - \frac{\delta g}{\delta \Phi}\frac{\delta f}{\delta Q} \tag{8}$$

$$\rightarrow \{\Phi, Q\} = \frac{\delta \Phi}{\delta \Phi}\frac{\delta Q}{\delta Q} - \frac{\delta Q}{\delta \Phi}\frac{\delta \Phi}{\delta Q} = 1 - 0 = 1 \tag{9}$$

the quantum operators similarly satisfy a commutation relation:

$$[\hat{\Phi}, \hat{Q}] = \hat{\Phi}\hat{Q} - \hat{Q}\hat{\Phi} = i\hbar \tag{10}$$

Quantum LC Hamiltonian

Defining the quantum operators: reduced flux $\phi \equiv 2\pi\Phi/\Phi_0$ and the reduced charge $n = Q/2e$, the quantum-mechanical Hamiltonian for the circuit can be written:

$$H = 4E_C n^2 + \frac{1}{2}E_L\phi^2 \tag{11}$$

where $E_C = e^2/(2C)$ is the charging energy required to add each electron of the Cooper-pair to the island and $U_L = (\Phi_0/2\pi)^2 / L$ is the inductive energy, where $\Phi_0 = h/(2e)$ is the superconducting quantum magnetic flux .

The quantum operator n is the excess number of Cooper-pairs on the island, and ϕ – the reduced flux – is denoted the "gauge-invariant phase" across the inductor. These two operators form a canonical conjugate pair, obeying the commutation relation $[\phi, n] = i$.

Quantum LC Hamiltonian

The Hamiltonian is identical to the one describing a particle in a one-dimensional quadratic potential (elastic force). We can consider ϕ as the generalized position coordinate, so that the first term is the kinetic energy and the second term is the potential energy. The form of the potential energy influences the eigensolutions. In particular, since the potential energy term is quadratic ($U_L \propto \phi^2$), the solution to this eigenvalue problem gives an infinite series of eigenstates $|k\rangle, (k = 0, 1, 2, \dots)$, whose corresponding eigenenergies E_k are all equidistantly spaced,i.e.:

$$E_{k+1} - E_k = \hbar\omega_r$$

where

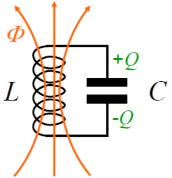
$$\omega_r = \sqrt{8E_L E_C/\hbar} = 1/\sqrt{LC}$$

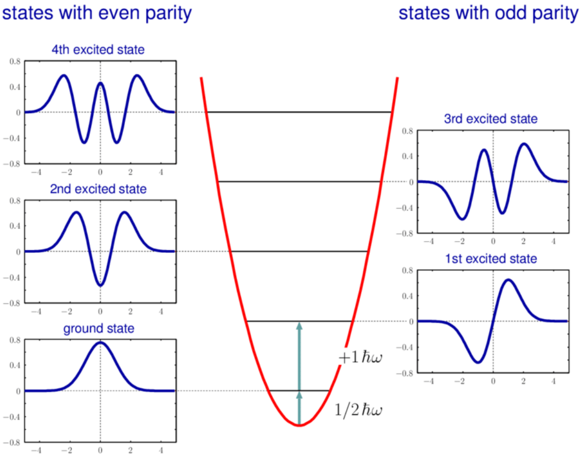
denotes the resonant frequency of the system.

A more compact form for the quantum harmonic oscillator (QHO) Hamiltonian is:

$$H = \hbar\omega_r \left(n + \frac{1}{2}\right) \tag{12}$$

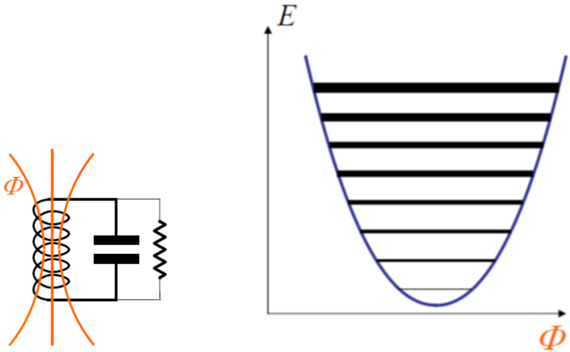
Dissipationless LC Circuit





Dissipationless LC Circuit

By adding a dissipative element in parallel (Resistor)



$$
\begin{aligned}
E_n &= \hbar\omega_r\left[n\left(1+\frac{i}{2Q}\right)+\frac{1}{2}\right] \\
\mathcal{Q} &= RC\omega_r
\end{aligned}
\tag{13}
$$

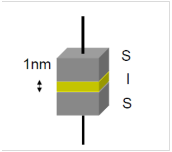
Dissipation broadens energy levels $\rightarrow$ undesidered !

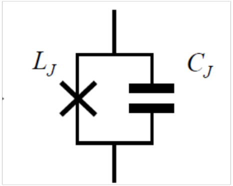
Limits of the Quantum Harmonic Oscillator

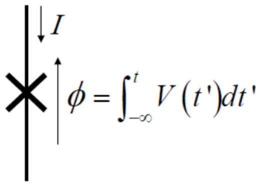
The linearity of the QHO is a limitation for processing quantum information. To be used as a qubit, a computational subspace consisting of only two energy states (usually the two-lowest energy eigenstates) in between which transitions can be driven without also exciting other levels in the system is required. Since many gate operations, such as single-qubit gates, depend on frequency selectivity, the equidistant level-spacing of the QHO poses a practical limitation.

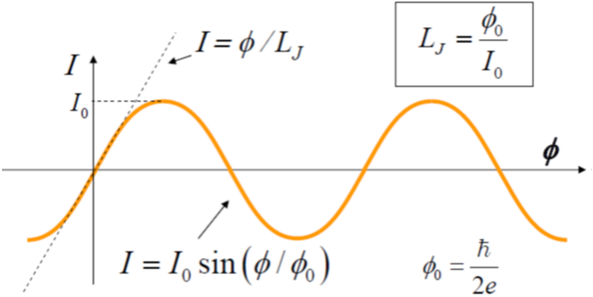
To mitigate the problem of unwanted dynamics involving non-computational states, we need to add anharmonicity (or nonlinearity) into our system. In short, we require the transition frequencies $\omega_{\mathrm{q}}^{0\to1}$ and $\omega_{\mathrm{q}}^{1\to2}$ to be sufficiently different to be individually adressable. In general, the larger the anharmonicity the better. In practise, the amount of anharmonicity sets a limit on how short the pulses used to drive the qubit can be.

Josephson Junction LC Circuit

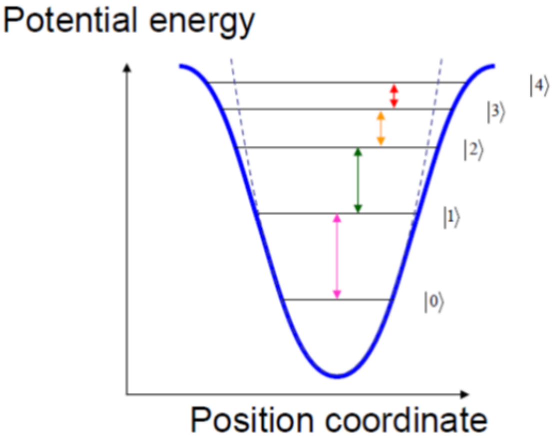








Josephson Junction LC Circuit



Anharmonic Josephson Junction Oscillator

We consider the Josephson junction – a nonlinear, dissipationless circuit element that forms the backbone in superconducting circuits. By replacing the linear inductor of the QHO with a Josephson junction, playing the role of a nonlinear inductor, we can modify the functional form of the potential energy. The potential energy of the Josephson junction can be derived from the Josephson relations of the current and

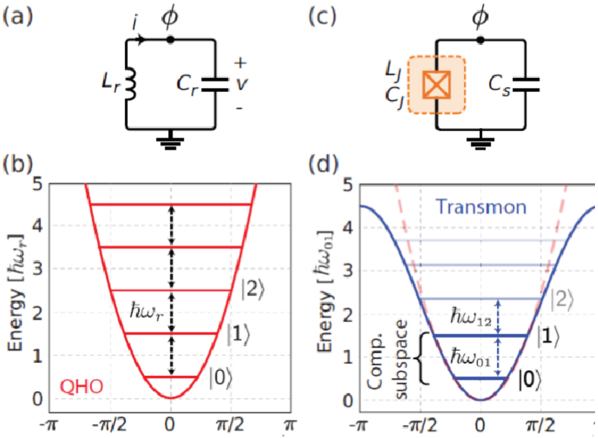
$$I = I_c \sin(\phi), \quad V = \frac{\hbar}{2e}\frac{d\phi}{dt} \tag{14}$$

resulting in a modified Hamiltonian

$$H = 4E_C n^2 - E_J \cos(\phi) \tag{15}$$

where $E_C = e^2/(2C_\Sigma)$, $C_\Sigma = C_s + C_J$ is the total capacitance, including both shunt capacitance C_s and the self-capacitance of the junction C_J, and $E_J = I_c\Phi_0/2\pi$ is the Josephson energy, with I_c being the critical current of the junction.
The potential energy takes a cosinusoidal form, which makes the oscillator anharmonic and the energy spectrum non-degenerate.

Anharmonic Josephson Junction Oscillator



Anharmonic mechanic Oscillator

Note that anharmonicity are found in the mechanical case (Morse potential)